Multi-phase Image Segmentation with the Adaptive Deformable Mesh

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DTU - det bli'r til noget

1. Why use explicit curves

Curves defining the segmentation boundaries can be represented in two ways: *explicit* (e.g. connected line segments, or *snakes* [1]) or *implicit* (e.g. level set method [2]). The advantage of *explicit* curves are:

- Multi-phases support naturally
- Optimized curves: Adapt the curves with multi-resolution to minimize storage
- Extract quantitative information (e.g. length, area), derive the solution directly, etc.
- Higher performance

2. The model of explicit curves

We proposed an image segmentation method using explicit curves. The curves are defined by set of edges in a triangle mesh. Our model enables integration over the whole domain.

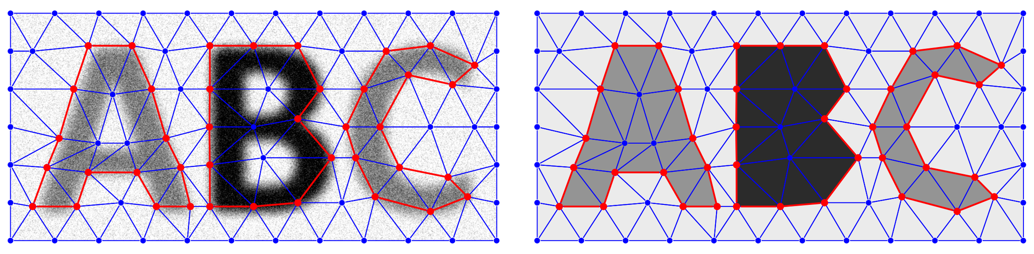


Figure 1: *A triangle mesh represents three-phase segmentation*

Connectivity changes are handled by employing Deformable Simplicial Complex [3] (**DSC**), an explicit interface tracking method.

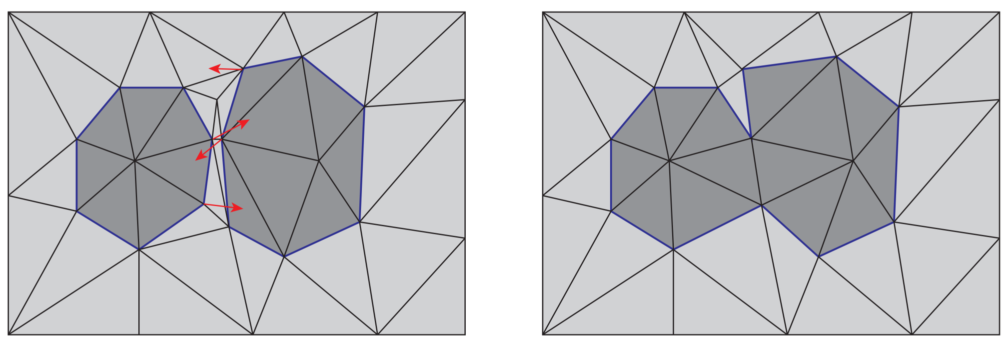


Figure 2: *Topology change in DSC during deformation: regions are merged when their interfaces collide. The red arrows show the displacements of the vertices.*

3. Method

Our method bases on minimizing the Mumford-Shah functional on the image. This functional defines a criterion for approximating an image $g : \Omega \to \mathbb{R}$ with a piecewise smooth function $u : \Omega \to \mathbb{R}$ and a boundary set $\Gamma \subset \Omega$. The energy function to be minimized is

$$E(u) = \sum_{n=1}^{N} \int_{\Omega_n} (c_n - g)^2 d\Omega + \alpha \, \text{length}(\Gamma), \qquad (1)$$

where α is the weight for the smoothness.

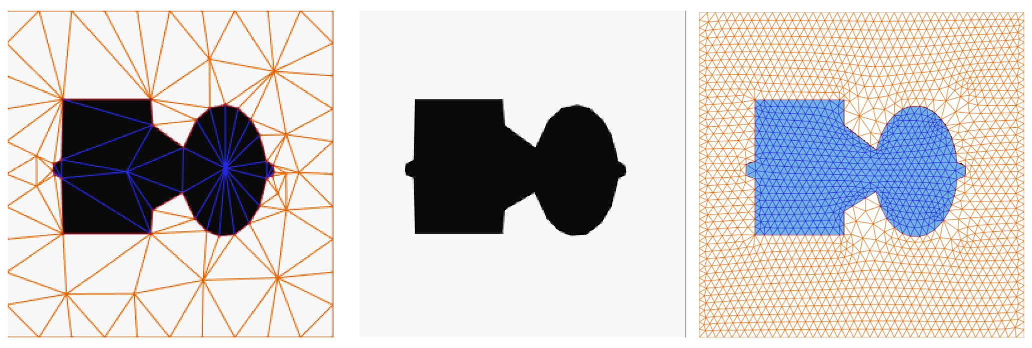
Applying gradient method, we derive the displacement of the interface vertices. Our algorithm is an iterative method given as follow

Algorithm 1: *Segmentation procedure*

while *residual error is still large* **do**
  Compute all interface-vertices' displacement
  Deform the mesh using **DSC**
  Adapt the mesh
end

4. Mesh adaptation

We adapt the mesh to have the optimal representation of the mesh, and to be able to capture all regions.



(a) Adaptive mesh (b) Output image (c) Dense mesh

Figure 3: *An adaptive mesh and a dense mesh achieving comparable segmentation. (a)* ~ 150 *triangles; (c)* ~ 3500 *triangles*

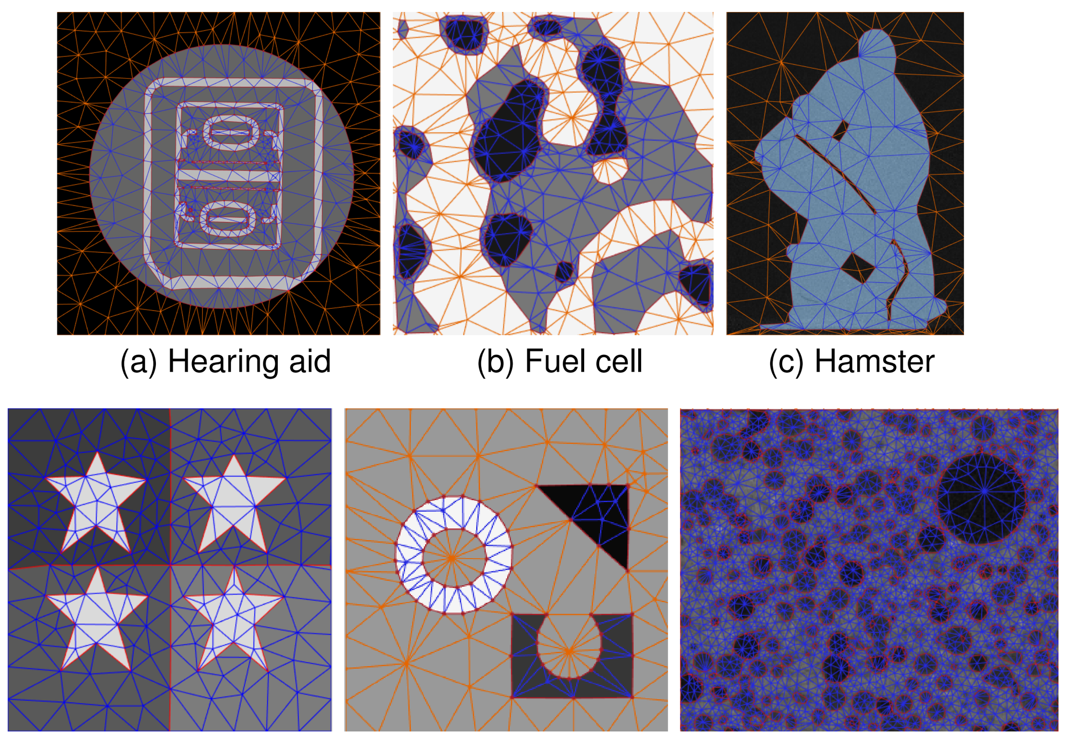
We propose two functions to compute energy on edges and triangles. These energies are the triggers to perform edge and triangle adaptation.

Algorithm 2: *Mesh adaptation*

```
// Split or collapse the edge base on its energy
```
Edge adaptation
```
// Split or change it label, depend on its energy
```
Triangle adaptation
```
// Remove vertex in homogeneous region
```
Thinning mesh

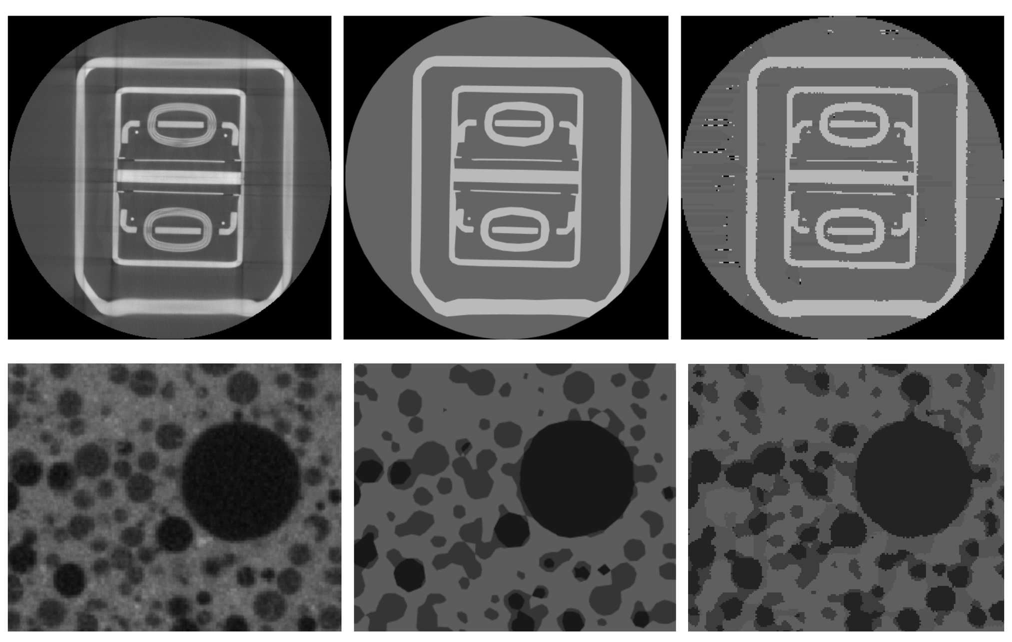
5. Results

Intensity base segmentation: We perform various segmentations and comparison to level set method



(a) Hearing aid (b) Fuel cell (c) Hamster
(d) Stars (e) Four phases (f) Cement

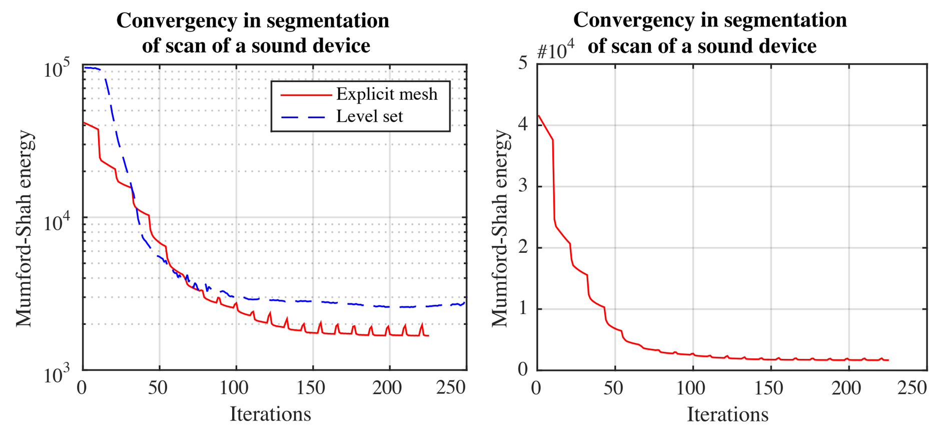
Figure 4: *Segmentation curves (red line) and the triangle mesh*



(a) Original image (b) Our method (c) Level set method

Figure 5: *Comparing our results with segmentation using implicit curve. First row: Scan of a hearing aid device. Second row: Scan of cement.*

Performance: depend on the size of the image, our method is up to 20 times faster than level set method.



(a) Semilog plot (b) Normal plot

Figure 6: *Mumford-Shah energy with respect to number of iterations in segmentation of scan of a hearing aid device.*

Extension: The algorithm can also be extended for other segmentation model (e.g. dictionary method) and 3D segmentation.

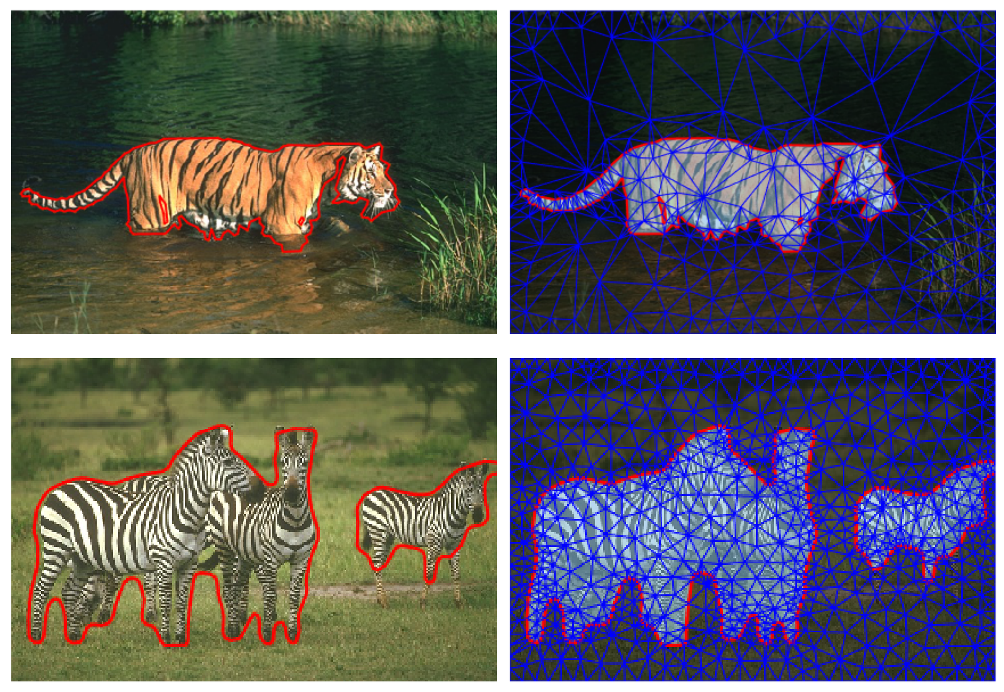


Figure 7: *Deformable mesh evolved by similarity of image patches*

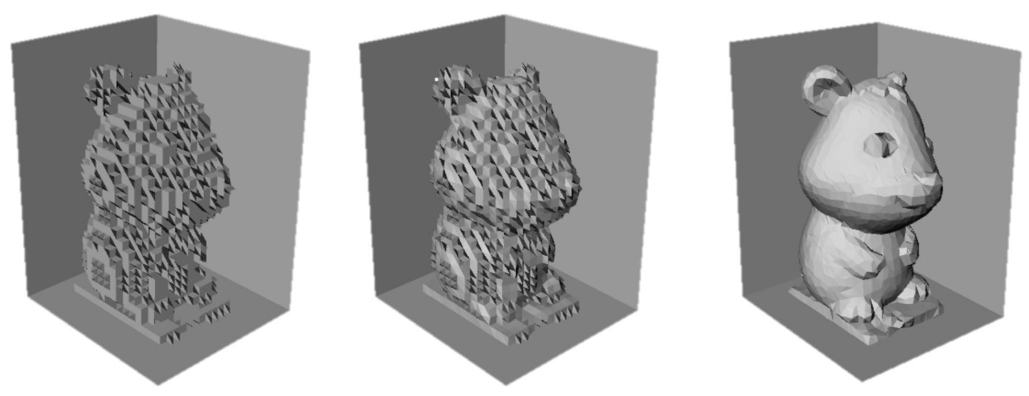


Figure 8: *3D segmentation from CT scan with deformable tetrahedral mesh*

6. Conclusions

We have proposed an algorithm for image segmentation using a deformable triangle mesh.
Advantages: We can segment an arbitrary number of phases; and the accuracy is high with an optimal representation of the mesh.
Limitation: Our method requires the user to select four parameters.

References

[1] M. Kass, A. Witkin, and D. Terzopoulos, "Snakes : Active Contour Models," *International Journal of Computer Vision*, vol. 331, no. 4, pp. 321–331, 1988.

[2] T. F. Chan and L. A. Vese, "Active contours without edges," *IEEE Transactions on Image Processing*, vol. 10, no. 2, pp. 266–277, 2001.

[3] M. K. Misztal and J. A. Bærentzen, "Topology-adaptive interface tracking using the deformable simplicial complex," *ACM Transactions on Graphics*, vol. 31, no. 3, pp. 1–12, may 2012.