A special limited-angle tomographic problem

Leise Borg, University of Copenhagen

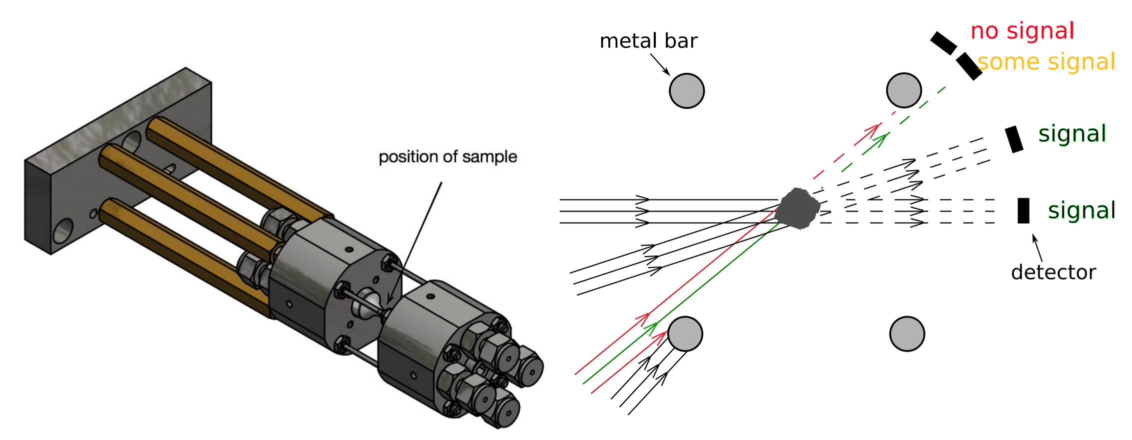
Supported by Maersk Oil and Gas
and Innovation Fund Denmark

Introduction

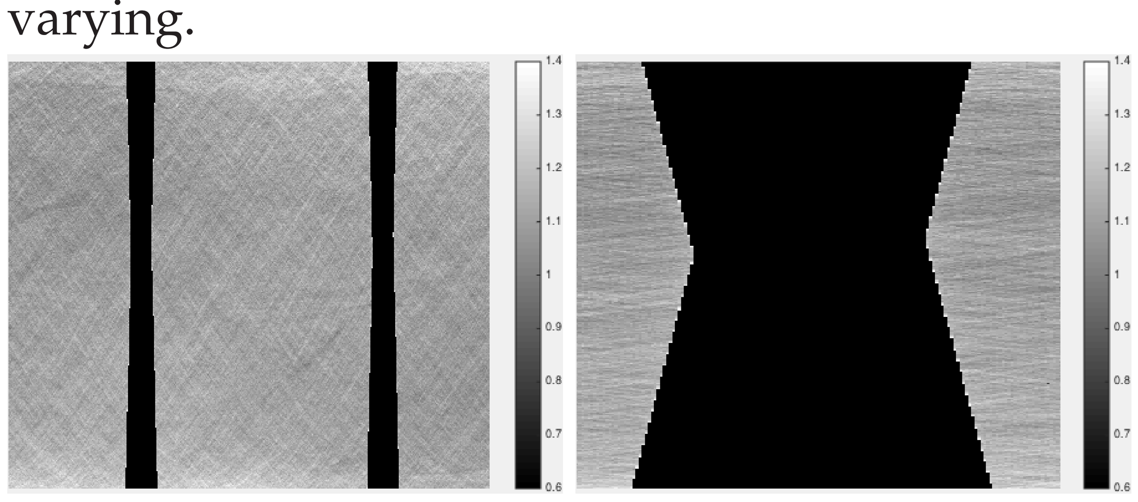
Modeling grain and pore structure in rock samples is useful in prediction of petrophysical parameters. Such models can be estimated from computed tomography images.

A special limited-angle problem

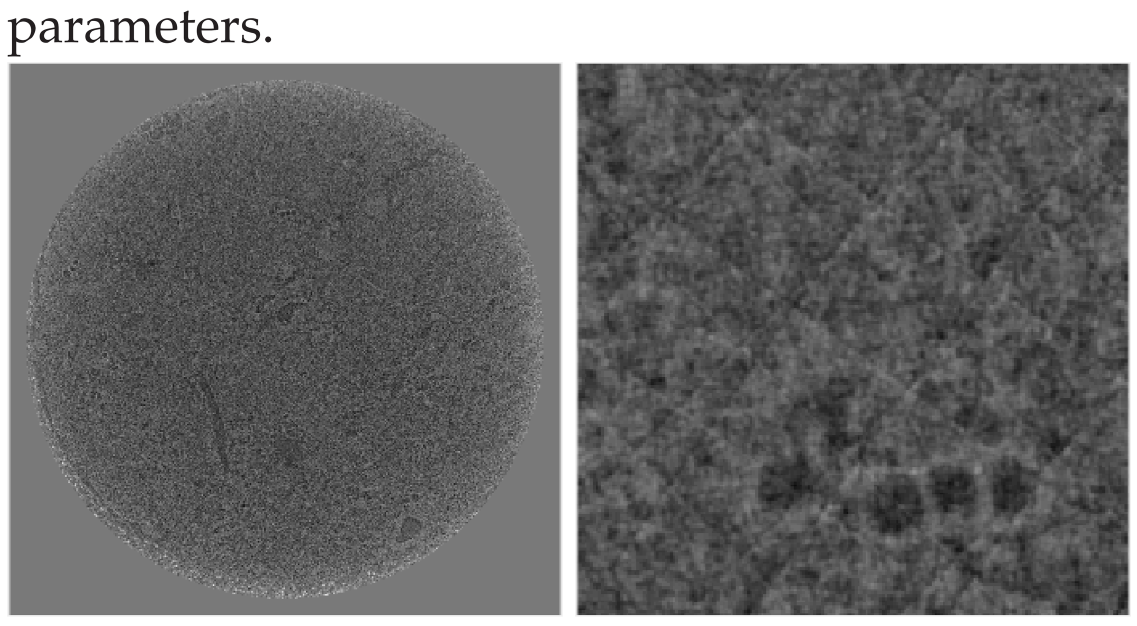
The experimental setup: A percolation cell is developed for performing in situ tomography of chalk samples subjected to a flow fluids through the pore space at high pressure. It is designed such that it has metal bars that shadow for the signal in certain directions. Further, for some angles, the effective detector width is decreased and incomplete signals are observed. This is illustrated in the situation "some signal" example in the figure below. We refer to this special data as **transition data**.



The sinogram resembles a regular limited-angle (LA) problem, but is not a pure LA-problem. The partial absence of recorded signal appear as a wedgy shape in the sinogram, instead of the rectangular ones observed in typical LA-problems. This is because the effective size of the detector is varying.



The reconstructions obtained by filtered backprojection (FBP) contain artefacts which are problematic in the later determination of the petrophysical parameters.



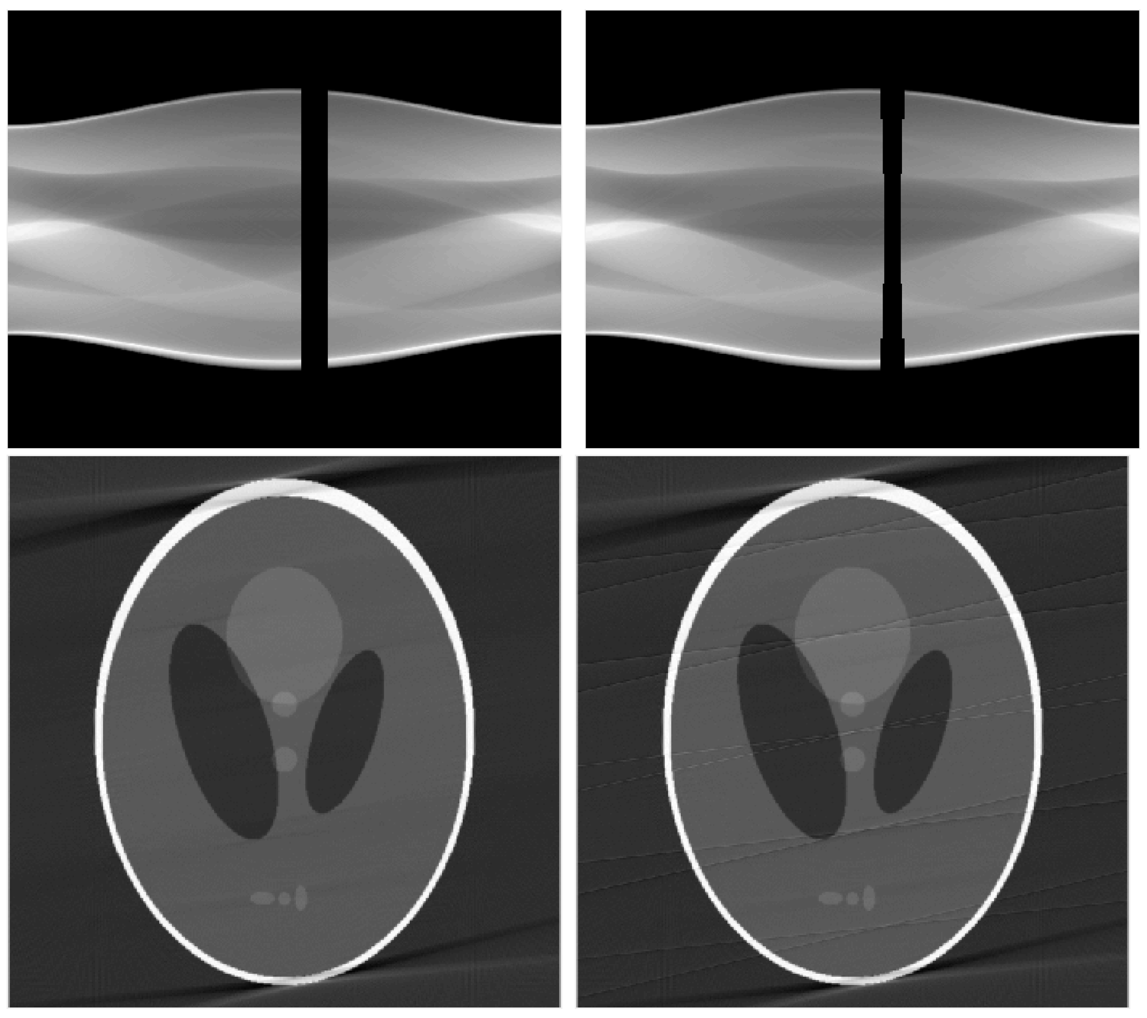
We observe that the spacing between the streaks is almost equal, and they appear at angles of approximately 45 degrees and 135 degrees.

To understand the artefacts in the reconstruction is crucial, such that we can reduce them while limiting image degradation.

Characterization of artefacts

To understand the artefacts in the reconstruction, we investigate a problem for the Shepp-Logan phantom.

In the figures below, a typical LA-sinogram and the corresponding reconstruction are shown in the first column. For this typical LA setup, the singularities only appear in the angular direction in the sinogram. In the second column the sinogram has been modified, such that a little more data is now present. This extra data does not fill in the entire sinogram column for that specific angle, but is truncated at specific positions - just as we observed in the real-data problem. This means that there are singularities in the angular (horizontal) *and* in the detector (vertical) directions. The corresponding reconstruction is shown in the bottom of the second column.



In the first reconstruction, we observe typical LA-artefacts: Streaks emerge from singularities in the reconstruction and have angles corresponding exactly to the singularities in the sinogram (note that the angles are shifted 90 degrees in the reconstruction). These artefacts are also present in the second reconstruction and will be ignored in this analysis from now. We observe following for the special LA-case (the second column):

In the sinogram	In the reconstruction
The singularities have en angle of 100 degrees $\pm \sim 5$ degrees	The streaks have en angle of 100 degrees $\pm \sim 5$ degrees.
There are 5 sets of singularities in the detector direction (there are also one set in the center, which cannot be seen in the figure)	There are 5 sets of streaks

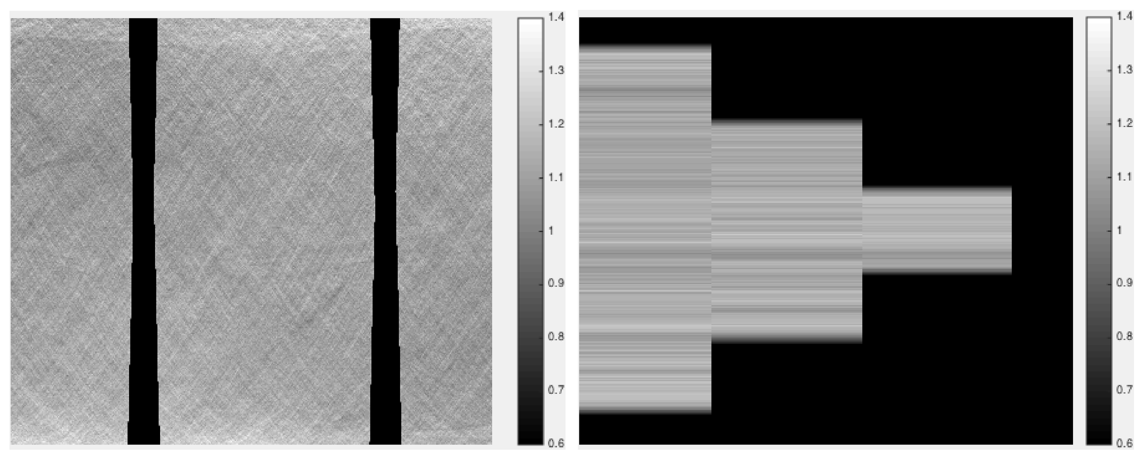
We conclude that the streaks come from the sinogram singularities - but only the ones in the detector direction.

Methods

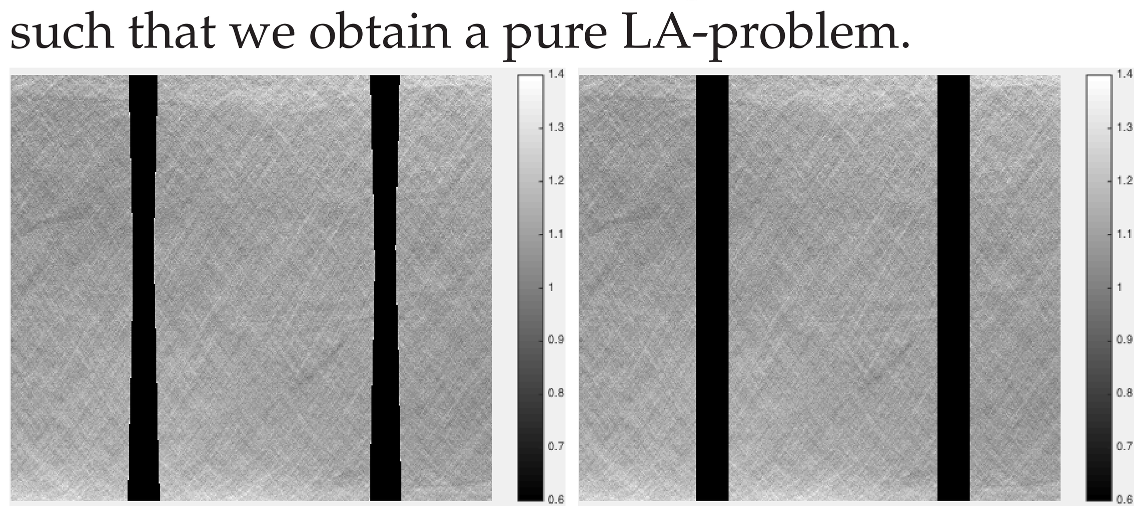
The theory of micro-local analysis proves that there exist a way to reduce artefacts arising from typical LA-problems: Smoothing the sinogram singularities arising from the LA-geometry [FQ13]. The intuition about this is that it is only possible to reconstruct singularities (edges) if the sinogram contains singularities. Therefore, if we remove the LA-singularities in the sinogram by smoothing them out, the singularities in the reconstruction - or the unwanted artefacts - get reduced. Since the LA-singularities exist (only) in the direction of the angular measurements, the smoothing is only applied in the angular direction.

Inspired by this theory, we want to investigate the effect by smoothing the edges of the chalk-data sinogram that causes the artefacts in the reconstruction. In our case - as argued for before - the artefacts arise from the singularities in the detector direction. Hence, we smooth the vertical singularities in the sinogram to see if this reduce the artefacts.

Method 1 consists of smoothing the singularities in the transition data that arise from the special acquisition geometry. We only smooth in the detector direction.



Method 2 consists of removing the transition data, such that we obtain a pure LA-problem.



For both of the above methods, we have eliminated the detector directional sinogram singularities.

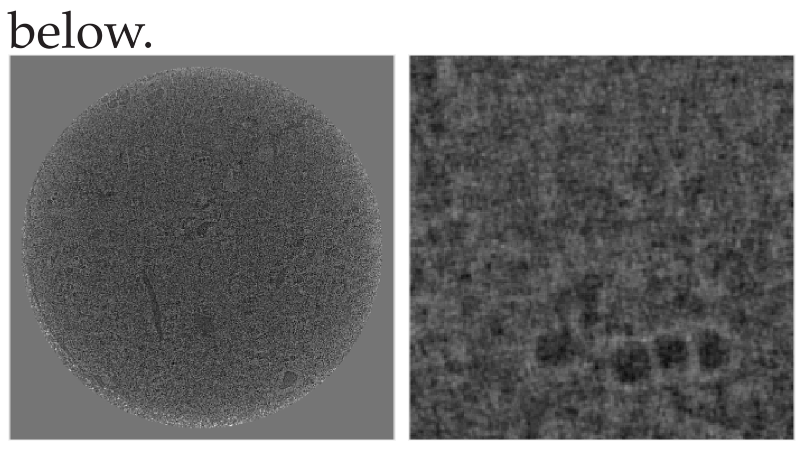
Conclusion

We conclude that singularities in the detector direction of the sinogram give rise to streaks in the reconstruction. If we remove those singularities in the sinogram, we also remove the streaks in the reconstruction. We suggest two methods for removal of the singularities:

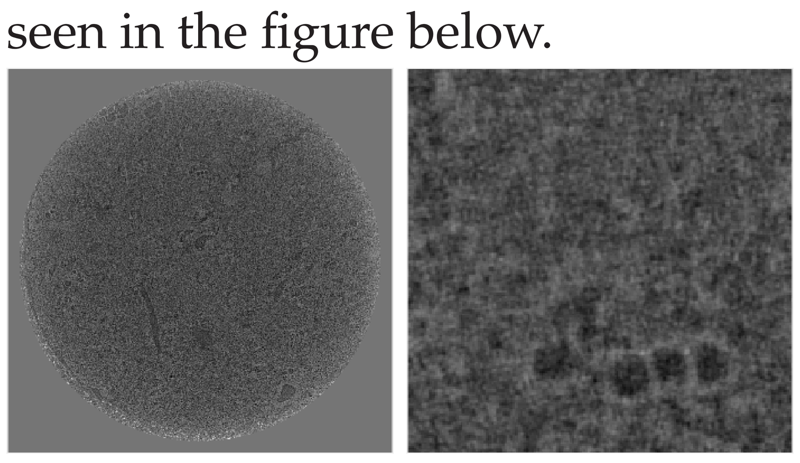
Method 1: Removal of the transition data.
Method 2: Smoothing the transition data.
This results in very similar reconstructions.

Results

Result 1: When smoothing the singularities in the detector direction, we also remove the artefacts in the reconstruction, as can be seen in the figure below.



Result 2: When removing a small portion of the data such that we avoid singularities in the detector direction, also removes the artefact, as can be seen in the figure below.



References

[FQ13] J. Frikel and E. T. Quinto. Characterization and reduction of artifacts in limited angle tomography. *Inverse Problems*, 29(12), 2013.