DTU Compute
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Probabilistic model based iterative CT

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Computed Tomography (CT) reconstructs images and volumes based on attenuation data. This task can be formulated as a probabilistic inference problem. This formulation allows for inclusion of accurate physical models of the attenuation physics and prior information available from other modalities such as MRI.
CT can also be performed using layered neural networks and may benefit from semi-supervised learning methods.

Introduction

In CT, an image of tissue attenuation coefficients is reconstructed from measurements of attenuated x-ray intensities along paths in the object.
Conventional CT methods such as Filtered Back Projection do not support inclusion of any detailed physical models of the signal acquisition.

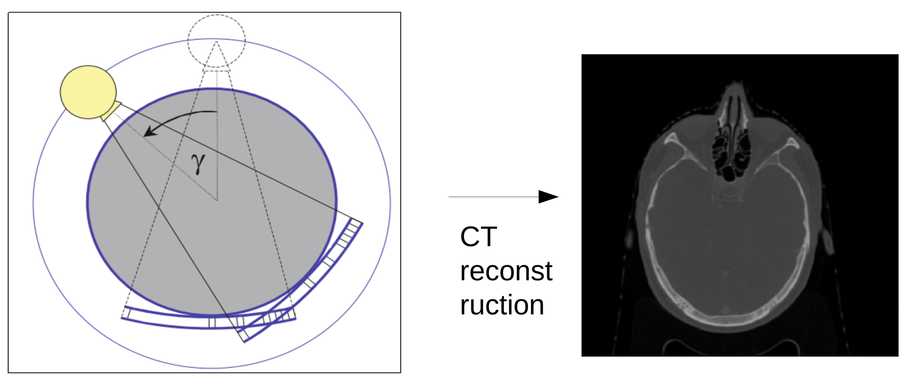


Illustration of CT. In most clinical scanners, a fan beam geometry is used. Using the detected intensities and knowledge of the initial intensity, the attenuation is measured and used to infer the distribution of attenuation coefficients in the image.

Attenuation physics

Conventionally, the x-rays are assumed monoenergetic and the attenuation simply removes x-rays from the beam at an energy independent rate.

There are at least 3 problems with this:

1): The source is not monochromatic and the attenuation coefficient is energy dependent.
2): Many photons are scattered rather than absorbed.
3): The detected signal depends on the energy of the photon.
Accurate attenuation and noise modelling is important particularly with metal artifacts, where energy dependent effects are amplified and the transmitted count is low.
These problems lead to streak and cupping artifacts.

MAP approach to CT

x-ray generation, attenuation and detection are stochastic so a probabilistic relationship between the image and measurements is preferred. Maximum a posteriori optimisation (MAP) can then be used to infer the image given the measurements.

The image (log)-likelihood contains a posterior and prior term:
L(μ) = log(P(n|μ)) + log(p(μ)),

Where n is a vector of measurements and μ is a vector of the image attenuation coefficients.

Probabilistic forward models (posterior term)

A probabilistic forward model is captured by the posterior probability distribution of the detected signal given the image.
Its shape depends on the detector model and the attenuation model.
Some possibilities are:

1. $P(n_i) = \frac{\lambda_i^{n_i} \mathrm{e}^{-\lambda_i}}{n_i!}$
   with $\lambda_i = \sum_{k=1}^{K} \Gamma_i(E_k) \exp(-\sum_{j=1}^{J} M_{i,j}\mu_{j,k})$.
2. $P(n_i) = \frac{\lambda^{n_i} \mathrm{e}^{-\lambda_i}}{n_i!}$
   with $\lambda_i(E_k) = \Gamma_i(E_k) T_i(E_k) = \Gamma_i(E_k) \exp(-\sum_{j=1}^{J} M_{i,j}\mu_{j,k})$.
3. $P(n_i) = \exp\left(-\sum_{k=1}^{K} \lambda_i(E_k)\right) \sum_{q=0}^{\infty} \frac{\lambda_i(\frac{n_i}{g})^{q*}}{q!}$
   with $\lambda_i(E_k) = \Gamma_i(E_k) T_i(E_k) = \Gamma_i(E_k) \exp(-\sum_{j=1}^{J} M_{i,j}\mu_{j,k})$.
4. $G_N(s_i) = \exp\left([-\sum_{k=1}^{K} \lambda(E_k) + \sum_{k=1}^{K} \lambda_i(E_k) \mathrm{e}^{gE_k(s_i-1)}]\right)$
   with $\lambda_i(E_k) = \Gamma_i(E_k) T_i(E_k) = \Gamma(E_k) \exp(-\sum_{j=1}^{J} M_{i,j}\mu_{j,k})$.

n_i: measured signal along I'th path
$m_{j,k}$: attenuation coefficient at energy k in voxel m
$M_{i,j}$: system matrix element, used to calculate discrete line integrals
$\lambda_{i,k}$: average transmitted count along path I of photons at energy k
Γ_k: emitted photons from source at energy k
G_N: Characteristic function of PDF

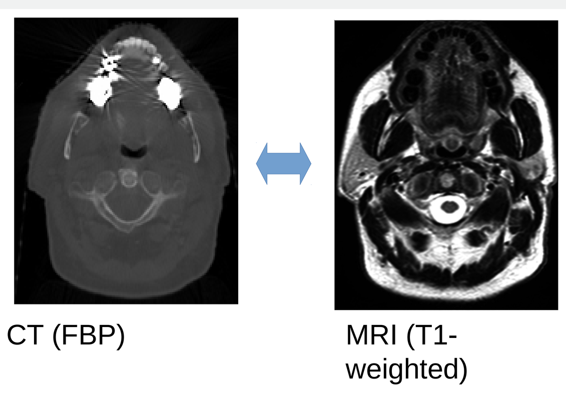
Probabilistic forward models for medical x-ray CT. 1) Poisson detector noise, monochromatic source. 2) Poisson detector noise, polychromatic source. 3) Energy dependent detection (compound Poisson noise) and polychromatic source. 4) Detection with scintillator crystal model, polychromatic source (characteristic function only)

Regularisation and complementary data (prior term)

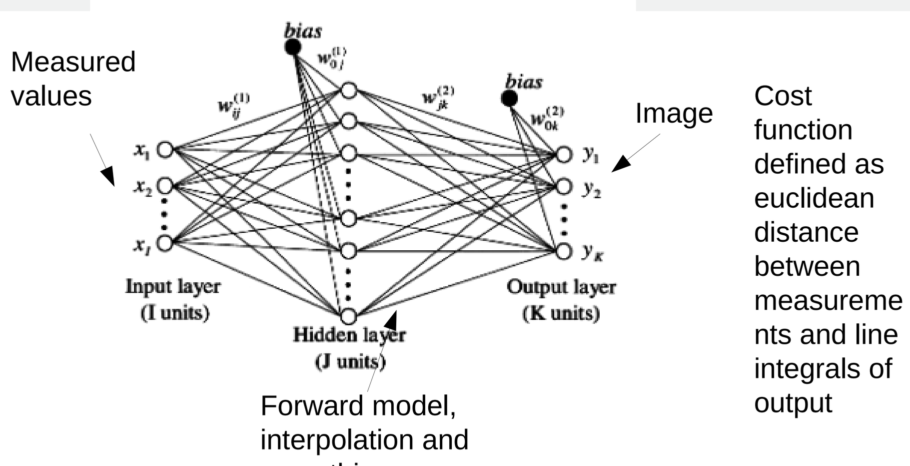
The a priori term of the image likelihood can be used to penalise roughness, sharp edges and noise in the reconstruction.
It can also be modified using for instance a coregistered MRI or a Pseudo-CT. MRI's have far more localised metal artifacts, making this feasible (see below).

CT and semi-supervised learning

CT can also be cast as a machine learning task. The images are the outputs and the measured signals are the inputs.



CT and MRI image of the same patient with dental fillings. The CT image is heavily contaminated by streak and cupping artifacts in regions close to the fillings. The Images are not perfectly coregistered and the slices are not exactly the same due to difference in the acquisition, but the observation holds for all slices in the neighborhood.



Multi layer network used effectively for reconstruction with incomplete data sets. Can deeper networks with unsupervised steps be used to include more of the statistical processes in the acquisition? To include MRI?