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LOCALLY ORDERLESS REGISTRATION OF DIFFUSION WEIGHTED IMAGES

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Abstract

Registration of Diffusion Weighted Images (DWI) is challenging as the data is a composition of both directional and intensity information. In this work, the density estimation framework for image similarity, Locally Orderless Registration, is extended to include directional information. **We construct a spatio-directional scale-space formulation of marginal and joint density distributions between two DWI.** We examine the scale-space and illustrate the approach by affine registration.

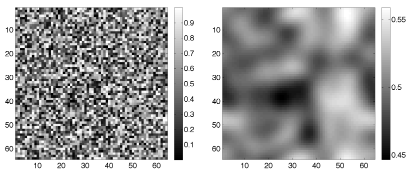
LOR

The LOR framework defines the similarity over 3 scales: The *image* scale, the *intensity* scale, and the *integration* scale.

Image scale

Given an image I, the first scale is the convolution with a kernel K_σ of standard deviation σ

$$I_\sigma(\boldsymbol{x}) = (I * K_\sigma)(\boldsymbol{x}) = \int_\Omega I(\boldsymbol{\tau})K_\sigma(\boldsymbol{x}-\boldsymbol{\tau})d\boldsymbol{\tau} \qquad (1)$$



(a) Original image (b) Smoothed image

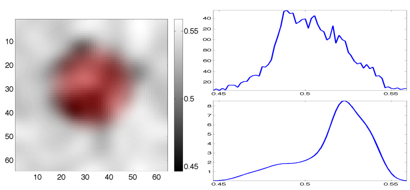
Figure 1: (Image scale) Examples of simple gaussian image smoothing.

Intensity and integration scale

To align two images I_σ and J_σ (1) under a transformation ϕ, we write up their joint histogram h. The intensity scale is essentially the soft bin width β of the histogram while the integration scale is a Gaussian integration window used to create a local histogram around $\boldsymbol{x}$ with standard deviation α.

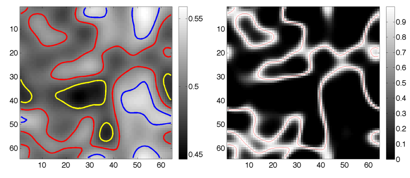
$$h_{\beta\alpha\sigma}(i,j|\phi,\boldsymbol{x}) = \int_\Omega \overbrace{P_\beta(I_\sigma(\phi(\boldsymbol{x}))-i)P_\beta(J_\sigma(\boldsymbol{x})-j)}^{\text{Intensity scale}}\ \overbrace{W_\alpha(\boldsymbol{\tau}-\boldsymbol{x})}^{\text{Integration scale}}\ d\boldsymbol{\tau} \qquad (2)$$

Here P_β is a Parzen-window, and W_α is a Gaussian integration window. This is a scale-space representation of intensity distributions in images and lets us use a set of generalized linear and non-linear similarity measures, such as Mutual Information (MI).



(a) Illustration of a local histogram (b) Histogram of (top) smoothed image in Fig. 1a and (bottom) local integration of 2a.

Figure 2: (Integration scale) Illustration of local integration.



(a) Example of 3 isophotes (b) Soft isophote (red line in a)

Figure 3: (Intensity scale) Examples of isophotes (i.e. bins in the histogram). 3a illustrates 3 bins while 3b shows the effect of smoothing a bin.

LOR-DWI

DWI are highly sensitive so invariant similarity measures (e.g. MI) are attractive. However, density estimation is complicated by directional information.

Adding the orientation scale

We get an *orientational* scale by adding an antipodally symmetric kernel Γ_κ, and substitute the image model in (1) to encompass a vector $\boldsymbol{v}$ on the spherical domain S^2

$$\begin{aligned} I_{\sigma\kappa}(\boldsymbol{x},\boldsymbol{v}) &= (I * (K_\sigma \otimes \Gamma_\kappa))(\boldsymbol{x},\boldsymbol{v}) \\ &= \int_{S^2}\left(\int_\Omega I(\boldsymbol{\tau},\boldsymbol{v})K_\sigma(\boldsymbol{\tau}-\boldsymbol{x})d\boldsymbol{\tau}\right)\overbrace{\Gamma_\kappa(\boldsymbol{v},\boldsymbol{v})}^{\text{Orientation scale}}\ d\boldsymbol{v} \end{aligned} \qquad (3)$$

We set Γ_κ to be a Watson distribution with concentration parameter κ.

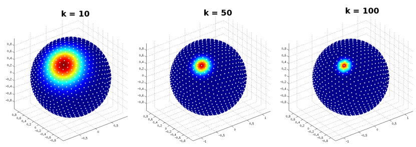


Figure 4: (Orientation scale) Example of a Watson distribution of varying support.

Now, let $\phi(\boldsymbol{x})$ be a diffeomorphic transformation of a point $\boldsymbol{x}$, $\psi(\boldsymbol{v}) = \frac{d\phi(\boldsymbol{v})}{|d\phi(\boldsymbol{v})|}$ be a deformation of the orientation $\boldsymbol{v}$ (based on the Jacobian of ϕ), and the overall transformation be $\tilde{\Phi} : (\boldsymbol{x},\boldsymbol{v}) \mapsto (\phi(\boldsymbol{x}),\psi(\boldsymbol{v}))$. Updating the formulation of (2) with (3), we get the joint histogram

$$h_{\beta\alpha\sigma\kappa}(i,j|\tilde{\Phi},\boldsymbol{x}) = \int_{\Omega\times S^2} P_\beta(I_{\sigma\kappa}(\phi(\boldsymbol{x}),\psi(\boldsymbol{v}))-i)P_\beta(J_{\sigma\kappa}(\boldsymbol{x},\boldsymbol{v})-j)W_\alpha(\boldsymbol{\tau}-\boldsymbol{x})d\boldsymbol{\tau}\times d\boldsymbol{v} \qquad (4)$$

and from this, the normalized density estimate

$$p_{\beta\alpha\sigma\kappa}(i,j|\tilde{\Phi},\boldsymbol{x}) \approx \frac{h_{\beta\alpha\sigma\kappa}(i,j|\tilde{\Phi},\boldsymbol{x})}{\int_{\Lambda^2} h_{\beta\alpha\sigma\kappa}(i,j|\tilde{\Phi},\boldsymbol{x})dl\,dk} \qquad (5)$$

The marginal probabilities are trivially derived, and from this we can use linear (sum of squared differences, Huber, ...) and non-linear measures (MI, NMI, ...) for image registration.

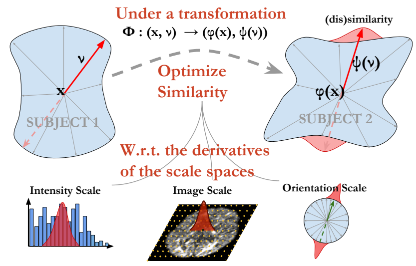


Figure 5: 2D illustration of a vector in a DWI shell (subject 1) being compared with another shell (subject 2). We calculate the derivatives of the diffeomorphic transformation w.r.t. the similarity over the smoothed scale spaces.

Experiments & Results

We performed a non-rigid registration of 2 high angular DWI scans (HARDI) from the Human Connectome Project (single shell, b=1000). We show that, even without any regularization term, the inherent information in the micro-structure adds its own regularization. The acquired DWI were pre-registered spatially to an MNI152 template, no global affine transformation was performed, and 30 (out of 90) gradient directions were used. Below, Figure 6 and 7 show an axial and a coronal slice from the registration. The images are the interpolated mean diffusivity.

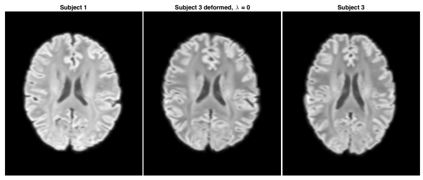


Figure 6: Interpolated mean diffusivity (axial slice). To the right (Subject 1) is the source image that the left target image (Subject 3) is registered to. The center image shows the non-rigid transformation of the target image.

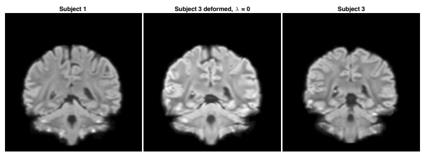


Figure 7: Interpolated mean diffusivity (coronal slice). Same as in Figure 6.

Benefits of LOR-DWI?

We extended the LOR density estimation framework to DWI. Our LOR-DWI framework allows us

- **Explicit control over the 4 scales of DWI (image scale, intensity scale, integration scale, and orientation scale).**

The scale-space formulation enables us to

- **Compare DWI across different resolutions.**

By adding a symmetric kernel on the sphere and including the first-order information of the diffeomorphic deformation, we

- **Account for the local geometrical transformation prior to modelling the distribution.**

Invariant similarity measures a key to handling the sensitive DWI scans and using our framework we can

- **Project complex DWI structures down to a 2D density estimate that enables the use of well-defiend similarity measures, such as Mutual Information.**

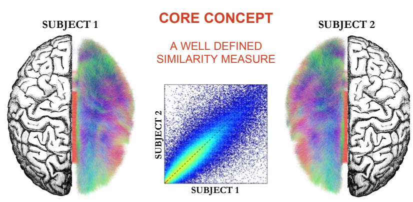


Figure 8: Illustration of two complex DWI structures projected down to a 2D histogram (density estimate) using LOR-DWI.